

Issues on magnon reflection

L. Palla^{*}

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Institute for Theoretical Physics

Eötvös University

H-1117 Budapest, Pázmány P. sétány 1 A, Hungary

Abstract

Two questions related to reflections of magnons in AdS/CFT are discussed: namely the problem of explaining the (physical) poles of the reflection amplitudes using Landau type diagrams and the generalization of the Ghoshal-Zamolodchikov boundary state formalism to magnon reflections.

^{*}palla@ludens.elte.hu

1 Introduction

The recently discovered integrable structures [1] in the planar $\mathcal{N} = 4$ super Yang-Mills theory make possible to test the AdS/CFT correspondence [2] over the entire range of the 't Hooft coupling. The scaling dimensions of ‘infinitely long’ single trace operators are computable to all orders in perturbation theory by mapping the dilatation operator to an integrable spin chain [3]. The spectrum is compared to the energy spectrum of a free closed string moving in $\text{AdS}_5 \times S^5$ with large angular momentum. In this limit the physical content of the theory is the spectrum of asymptotic states and their scattering matrix and they are highly constrained by the residual symmetries. Indeed Beisert showed [4] that the residual symmetries are sufficient to determine the two particle scattering matrix between the elementary excitations (known as magnons) up to an overall scalar factor. This overall scalar factor, known as “dressing factor” plays an important role in interpolating the weak-strong coupling spectrum of the gauge/string correspondence [5], [6], [7]. The dressing factor obeys an extra symmetry known as “crossing” imposed on the two particle scattering matrix [6], [7], [8].

Recently Hofman and Maldacena [9] considered open strings attached to maximal giant gravitons [10] in $\text{AdS}_5 \times S^5$ (for related earlier work see [11]). They determined reflection amplitudes (sometimes also called boundary S matrices) of the elementary magnons for two (integrability preserving) cases, namely the $Y = 0$ and the $Z = 0$ giant graviton branes. For both cases the reflection amplitudes of the elementary magnons were determined up to an overall phase factor in [9] by exploiting the residual symmetries of the problem. This phase factor for the $Y = 0$ brane has recently been determined in [12] from an analysis of the boundary crossing condition (BCC). The BCC for the $Z = 0$ brane has also been determined in [12] but the actual solution for the missing phase factor of the elementary magnon reflection was given in [13]. Using the fusion method the authors of [13] also determined the reflection amplitudes for all the magnon bound states for both the $Y = 0$ and the $Z = 0$ branes. These results are in a certain sense summarized in [14] where the authors extend the Zamolodchikov-Faddeev algebra for open strings attached to giant gravitons.

In this paper two questions related to magnon reflections are investigated. The first concerns the interpretation of certain poles of the reflection amplitudes in terms of the generalization of Landau diagrams. Some of the first order poles (for the $Z = 0$ brane) signal the presence of boundary bound states [9], [13], but not all the poles of the various reflection amplitudes admit this interpretation and one of the aims of this paper is to suggest

a possible explanation for these ‘no boundary bound state’ poles. This investigation is a generalization to the boundary case of the recent program aimed to use the generalization of the (bulk) Coleman-Thun mechanism to explain the various (physical) poles in the scattering of elementary magnons [15] and magnon bound states [16].

The second question we investigate is the generalization to magnon reflections of the boundary state formalism originally worked out for relativistic boundary integrable theories in [17]. The motivation for this investigation is at least twofold: on the one hand this way a new derivation of the boundary crossing condition (BCC) is obtained while on the other this formalism naturally connects the magnon reflection problem to the bulk mirror magnon theory introduced in [18].

The paper is organized as follows: in section 2 the basic facts about magnons are reviewed. In section 3 the poles in the magnon reflection amplitudes on the $Y = 0$ brane are exhibited. Section 4 is devoted to the generalization of the boundary Coleman-Thun mechanism for the magnon problem; first some general remarks are made (valid also for magnon scattering in the bulk) then the search for the appropriate Landau diagrams is described in details. The generalization of boundary state formalism for magnon reflections is described in section 5. We make our conclusions in sect. 6.

2 Some basic facts about magnons

2.1 Kinematics and spectrum

The fundamental excitations of the spin chain are the magnons, which form a sixteen-dimensional (short) BPS representation of the unbroken $SU(2|2) \times SU(2|2)$ supersymmetry. The closure of the SUSY algebra on this multiplet uniquely determines [4] the magnon dispersion relation [19] (see also [20]),

$$E = \Delta - J = \sqrt{1 + 16g^2 \sin^2 \left(\frac{p}{2} \right)} \quad (2.1)$$

where $g = \sqrt{g_{YM}^2 N}/4\pi$. It is convenient to describe the magnons in terms of two complex spectral parameters x^\pm . In terms of these parameters, the magnon momenta and energies

are expressed as,

$$p = p(x^\pm) = \frac{1}{i} \log \left(\frac{x^+}{x^-} \right), \quad (2.2)$$

$$\epsilon = \epsilon(x^\pm) = \frac{g}{i} \left[\left(x^+ - \frac{1}{x^+} \right) - \left(x^- - \frac{1}{x^-} \right) \right]. \quad (2.3)$$

The dispersion relation (2.1) is equivalent to the constraint

$$\left(x^+ + \frac{1}{x^+} \right) - \left(x^- + \frac{1}{x^-} \right) = \frac{i}{g}. \quad (2.4)$$

Furthermore, any number of elementary magnons can form a stable bound state. The Q -magnon bound state ($Q \in \mathbb{N}$) also belongs to a BPS representation of supersymmetry (of dimension $16Q^2$). Therefore there is an infinite tower of BPS states labeled by a positive integer Q in the theory. The exact dispersion relation for these states is again fixed by supersymmetry to have the form [21, 22],

$$E = \sqrt{Q^2 + 16g^2 \sin^2 \left(\frac{P}{2} \right)} \quad (2.5)$$

The spectral parameters of the constituent magnons in a Q -magnon bound state are:

$$x_j^- = x_{j+1}^+ \quad \text{for } j = 1, \dots, Q-1. \quad (2.6)$$

The resulting bound state is described by the spectral parameters

$$X^+ \equiv x_1^+, \quad X^- \equiv x_Q^-, \quad (2.7)$$

and the total momentum P and $U(1)$ charge Q of the bound state are expressed as

$$P(X^\pm) = \frac{1}{i} \log \left(\frac{X^+}{X^-} \right), \quad (2.8)$$

$$Q(X^\pm) = \frac{g}{i} \left[\left(X^+ + \frac{1}{X^+} \right) - \left(X^- + \frac{1}{X^-} \right) \right]. \quad (2.9)$$

One can also show the energy $E = \sum_{k=1}^Q \epsilon_k$ for the bound state is related to the spectral parameters X^\pm through the expression

$$E(X^\pm) = \frac{g}{i} \left[\left(X^+ - \frac{1}{X^+} \right) - \left(X^- - \frac{1}{X^-} \right) \right], \quad (2.10)$$

while in terms of P and Q ,

$$E(P; Q) = \sqrt{Q^2 + 16g^2 \sin^2 \left(\frac{P}{2} \right)}. \quad (2.11)$$

The velocity of the particle in appropriately-normalized world sheet coordinates (x, t) is given as

$$v(X^\pm) = \frac{dx}{dt} = \frac{1}{2g} \frac{dE}{dP} = \frac{X^+ + X^-}{1 + X^+ X^-} = \frac{2g \sin(P)}{\sqrt{Q^2 + 16g^2 \sin^2\left(\frac{P}{2}\right)}}. \quad (2.12)$$

2.2 Bulk S matrix

In the infinite asymptotic spin chain limit the elementary magnons propagate freely apart from pairwise scattering described by the two body scattering matrix $S(x_1, x_2)$. It was shown by Beisert [4], that by demanding the invariance of $S(x_1, x_2)$ under the symmetry algebra it can be constrained up to an overall scalar factor. Various versions of this S matrix are in use in the literature depending on the choice of a basis. Here we use the so called “string” basis [23] where

$$S_{\text{full}} = S_0^2(x_1, x_2) (\hat{S}_{su(2|2)}(x_1, x_2) \otimes \hat{S}_{su'(2|2)}(x_1, x_2)). \quad (2.13)$$

The flavour dependent $\hat{S}_{su(2|2)}(x_1, x_2)$ and $\hat{S}_{su'(2|2)}(x_1, x_2)$ parts are uniquely determined by the symmetry algebra and non trivially satisfy the Yang-Baxter and unitarity equations. The scalar factor is given by [23]

$$S_0^2(x_1, x_2) = \frac{(x_1^- - x_2^+)}{(x_1^+ - x_2^-)} \frac{(1 - \frac{1}{x_1^+ x_2^-})}{(1 - \frac{1}{x_1^- x_2^+})} \sigma^2(x_1^\pm, x_2^\pm) \equiv \frac{\tilde{S}(x_1^\pm, x_2^\pm)}{A(x_1^\pm, x_2^\pm)}, \quad A(x_1^\pm, x_2^\pm) = \frac{(x_1^+ - x_2^-)}{(x_1^- - x_2^+)}, \quad (2.14)$$

where $\sigma^2(x_1^\pm, x_2^\pm)$ is usually referred to as the “dressing factor”. The conjectured exact expression for this function [7] is conveniently given as an integral representation [15],

$$\sigma(x_1^\pm, x_2^\pm) = \left(\frac{R(x_1^+, x_2^+) R(x_1^-, x_2^-)}{R(x_1^+, x_2^-) R(x_1^-, x_2^+)} \right), \quad R(x_1, x_2) = e^{i[\chi(x_1, x_2) - \chi(x_2, x_1)]}, \quad (2.15)$$

where

$$\chi(x_1, x_2) = -i \oint_C \frac{dz_1}{2\pi} \oint_C \frac{dz_2}{2\pi} \frac{\log \Gamma \left(1 + ig \left(z_1 + \frac{1}{z_1} - z_2 - \frac{1}{z_2} \right) \right)}{(z_1 - x_1)(z_2 - x_2)}, \quad (2.16)$$

with the contours in (2.16) being unit circles $|z_1| = |z_2| = 1$.

Now consider the highest weight $((1, 1))$ component of the magnon $\phi(x_1)$, that scatters diagonally, i.e. for which the full scattering amplitude can be written as

$$|\phi(x_1)\phi(x_2)\rangle = A(x_1^\pm, x_2^\pm) \tilde{S}(x_1^\pm, x_2^\pm) |\phi(x_2)\phi(x_1)\rangle. \quad (2.17)$$

(Note that - apart from the $\sigma^2(x_1^\pm, x_2^\pm)$ factor - the product $A(x_1^\pm, x_2^\pm) \tilde{S}(x_1^\pm, x_2^\pm)$ is nothing but the BDS piece of the S-matrix $S_{\text{BDS}}(x_1^\pm, x_2^\pm)$). Then, because of factorization of the multiparticle S-matrix, the two body scattering matrix between magnon bound states $\Psi_{Q_1}(X_1)$,

$\Psi_{Q_2}(X_2)$ with spectral parameters X_1^\pm and X_2^\pm , consisting of Q_1 and Q_2 pieces of ϕ respectively ($Q_1 \geq Q_2$), is simply obtained as the product of two body S-matrices describing all possible pair-wise scattering between the constituent magnons (“fusion procedure”). The outcome is

$$|\Psi_{Q_1}(X_1)\Psi_{Q_2}(X_2)\rangle = A(X_1^\pm, X_2^\pm)\tilde{S}(X_1^\pm, X_2^\pm) \prod_{k=0}^{Q_2-1} F(X_1^\pm, X_2^\pm, k) |\Psi_{Q_2}(X_2)\Psi_{Q_1}(X_1)\rangle, \quad (2.18)$$

with

$$F(X_1^\pm, X_2^\pm, k) = \left(\frac{X_1^+ + \frac{1}{X_1^+} - X_2^+ - \frac{1}{X_2^+} + \frac{ik}{g}}{X_1^- + \frac{1}{X_1^-} - X_2^- - \frac{1}{X_2^-} - \frac{ik}{g}} \right)^{2-\delta_{k,0}}. \quad (2.19)$$

(This form of $F(X_1^\pm, X_2^\pm, 0)$ is valid for $Q_1 > Q_2$; if $Q_1 = Q_2$ then $F(X_1^\pm, X_2^\pm, 0) \equiv 1$). The scattering phases appearing in eq.(2.17, 2.18) have first and second order poles. Some of the poles (like the first order ones at $X_1^- = X_2^+$) signal the possibility of forming bound states but not all of them can be explained this way and a program to explain them in terms of the particles in the spectrum using the generalization of Landau diagrams was initiated in [15] [16].

2.3 Physicality conditions

In general the singularities of the bulk S matrix or the reflection amplitudes occur at complex values of the external momenta and energies. Of these singularities only the ones in the ‘physical domain’ require an explanation in terms of the particles (and boundary bound states) in the spectrum. In ref.[15][16] the following condition was proposed to decide the ‘physicality’ of a bulk singularity: it is physical if parametrically it comes close to the positive real energy axis in any of the following three limits:

(i) The Giant Magnon limit: $g \rightarrow \infty$ while P kept fixed, when

$$X^+ \simeq 1/X^- \simeq e^{iP/2}, \quad E \simeq 4g \sin\left(\frac{|P|}{2}\right). \quad (2.20)$$

(ii) Plane-Wave limit: $g \rightarrow \infty$ with $k \equiv 2gP$ kept fixed, when

$$X^+ \simeq X^- \simeq \frac{Q + \sqrt{Q^2 + k^2}}{k} \in \mathbb{R}, \quad E \simeq \sqrt{Q^2 + k^2}. \quad (2.21)$$

(iii) Heisenberg spin-chain limit: $g \ll 1$ limit, when

$$X^\pm \mp \frac{iQ}{2g} \simeq \frac{1}{2g} \cot\left(\frac{P}{2}\right), \quad E \simeq Q + \frac{8g^2}{Q} \sin^2\left(\frac{P}{2}\right). \quad (2.22)$$

In the following we accept this condition also for singularities in the reflection amplitudes.

3 (Multi)magnon reflections

In integrable field theories new phenomena appear when they are restricted to a half line with some non trivial boundaries, the boundary sine-Gordon model being a well studied prime example [17]. Usually the integrability of the bulk theory is preserved for some special boundary conditions only. A similar thing happens in the AdS/CFT correspondence, where on the string theory side D-branes introduce non trivial boundaries of the string world sheet, while in the $\mathcal{N} = 4$ super Yang-Mills side (sub)determinant fields introduce boundaries to composite operators. Recently, in [9], two special (integrable) boundary conditions of the open spin chains in $\mathcal{N} = 4$ super Yang-Mills were investigated which describe giant gravitons interacting with the elementary magnons of the chain attached to it. The first case, the $Y = 0$ brane is represented by composite operators containing a determinant factor $\det(Y)$, while the second, the $Z = 0$ brane is represented by composite operators containing $\det(Z)$. (W, Y, Z denote the three complex scalar fields of the $\mathcal{N} = 4$ super Yang-Mills). The essential difference between the two cases is that in the $Z = 0$ brane the open super Yang-Mills spin chain is attached to the giant graviton through some boundary impurities $\chi, \bar{\chi}$, and as a result it has a boundary degree of freedom, while the $Y = 0$ brane has no boundary degrees of freedom.

3.1 Reflections on the $Y = 0$ brane

We start by reviewing the (multi)magnon reflection amplitudes on the $Y = 0$ brane. The reflection amplitude for the elementary magnon component ϕ (which is a singlet under the residual $su(1|2) \otimes su(1|2)$ symmetry) is given by

$$R_{R \text{ full}}^Y : |\phi(x^\pm)\rangle = -\sigma(x^\pm, -x^\mp)|\phi(-x^\mp)\rangle, \quad R_{L \text{ full}}^Y : R_{R \text{ full}}^Y(x^\pm \rightarrow -x^\mp) \quad (3.1)$$

for reflections on the left (respectively right) boundaries. Since they are obtained from each other by the parity transformation ($x^\pm \rightarrow -x^\mp$) in the following we concentrate on reflections from the right boundary only. For a magnon bound state $\Psi_Q(X)$ the fusion procedure (that now involves also the reflection of elementary magnons (3.1)) gives the total

reflection amplitude $R_{R \text{ full}}^Y$ as

$$|\Psi_Q(X^\pm)\rangle = R_{QR}(X)|\Psi_Q(-X^\mp)\rangle, \quad (3.2)$$

where

$$R_{QR}(X) = -\sigma^{-1}(-X^\mp, X^\pm) \prod_{k=1}^{Q-1} \left(\frac{X^+ + \frac{1}{X^+} - \frac{ik}{2g}}{X^- + \frac{1}{X^-} + \frac{ik}{2g}} \right). \quad (3.3)$$

Using the representation (2.15,2.16) one can show that $R_{QR}(X)$ has only first order poles and zeroes given by the following expressions

$$\text{poles at } X^- + \frac{1}{X^-} = i \frac{m}{2g} \quad m = -(Q-1), \dots, -1, 1, 2, \dots \quad (3.4)$$

$$\text{zeroes at } X^+ + \frac{1}{X^+} = i \frac{l}{2g} \quad l = (Q-1), \dots, 1, -1, -2, \dots, \quad (3.5)$$

where, for both poles and zeroes, the second (infinitely long) set originates from the dressing factor. It was remarked in [13] that the first order poles in (3.4) can not be identified with the formation of boundary bound states as they would not solve the boundary Bethe-Yang equation. Part of the aims of this paper is to outline an alternative explanation of these first order poles.

To check the ‘‘physicality’’ of the poles in (3.4) we determine X^\pm for the poles by combining (2.9) and (3.4):

$$X^+ = \frac{i}{4g} \left(2Q + m + \sqrt{16g^2 + (2Q + m)^2} \right) \quad (3.6)$$

$$X^- = -\frac{i}{4g} \left(\sqrt{16g^2 + m^2} - m \right). \quad (3.7)$$

Using them in (2.10) gives the energy of the Q magnon bound state at the pole as

$$E = \frac{1}{2} \left(\sqrt{16g^2 + (2Q + m)^2} + \sqrt{16g^2 + m^2} \right) \longrightarrow 4g \quad \text{for } g \rightarrow \infty, \quad (3.8)$$

while the momentum of the bound state (2.8), can conveniently be parameterized as $P = \pm\pi - iq$ with

$$q = \ln \frac{2Q + m + \sqrt{16g^2 + (2Q + m)^2}}{\sqrt{16g^2 + m^2} - m} \sim \frac{Q + m}{2g} \quad \text{for } g \rightarrow \infty. \quad (3.9)$$

These expressions show that in the giant magnon regime the pole satisfies the physicality condition. For later reference we also note that the velocity of the bound state at the pole can be written as

$$v(X) = \frac{1 + \frac{X^+}{X^-}}{X^+ + \frac{1}{X^-}} = -\frac{4g}{i} \frac{L}{N}, \quad L > 0, \quad N > 0, \quad (3.10)$$

indicating that it is indeed heading towards the right boundary.

4 Boundary Coleman-Thun mechanism for magnons

Explaining higher order poles in the exact S-matrices of integrable 1+1 dimensional models goes back to the work of Coleman and Thun [24], who were the first to realize that in 1+1 dimensions some anomalous thresholds may appear in the form of these poles. An interesting aspect of this analysis was the realization, that sometimes even first order poles may be explained as anomalous thresholds, since by this they broke the usual association of first order poles in the S-matrix with a particle state in either the forward or the crossed channel. In ref. [25] [26] and [27] it was found that this Coleman-Thun mechanism works also in the presence of (integrability preserving) boundaries, as several first order poles of the various reflection amplitudes - instead of describing boundary bound states - could be explained in terms of on-shell (anomalous threshold) diagrams for multiple scattering processes now involving also the reflections on the boundary*. It is important to emphasize that normally an anomalous threshold diagram would lead to a pole of order higher than one (for a diagram with N internal lines and L loops the order is $N - 2L$) and a first order pole is obtained either after taking into account the combination of several diagrams or because one or more “internal” reflection/scattering amplitudes develop zeroes exactly when the diagram goes on-shell.

One of the aims of this paper is to generalize the boundary Coleman-Thun mechanism for (multi)magnon reflections.

4.1 General remarks on Coleman-Thun mechanism for magnons

The Coleman-Thun mechanism is quantum field theoretic in nature as it relies on the Landau equations that determine the (necessary) conditions for a Feynman diagram to develop a singularity. To derive these equations [30] one assumes the usual (Minkowski space) form of the internal propagators in addition to conservation of energy and momentum at the internal vertices. Assuming that there is a (obviously non relativistic) field theory underlying the (multi)magnon scattering/reflections one may try to generalize the Landau equations for the magnon problem. In doing so one has to use the explicit form of the propagator on the

*The complete generalization of the underlying Landau equations for any (not necessarily integrable) relativistic boundary theory can be found in [28] [29]

internal lines. A natural choice is to take it in the form

$$\Pi(E, P) = \frac{i}{E^2 - 16g^2 \sin^2 \frac{P}{2} - Q^2 + i\epsilon} \quad (4.1)$$

for a magnon (bound state) with energy E and momentum P , since this is in accord with the dispersion relations (2.3, 2.11), and its denominator is also quadratic in the energy as in the relativistic case. Furthermore, after appropriate analytical continuations, this form may also describe the free propagator of the mirror magnon model [18]. Since energy and momentum are conserved at the vertices also for the magnon problem, using also (4.1) one can repeat the procedure in [30] with the outcome that the Landau equations for any diagram with I internal lines and L loops take the form[†]:

$$E_j^2 - 16g^2 \sin^2 \frac{P_j}{2} - Q_j^2 = 0, \quad j = 1, \dots, I, \quad (4.2)$$

for all the internal lines, and

$$\sum_{i \in L_l} \alpha_i E_i = 0, \quad -8g^2 \sum_{i \in L_l} \alpha_i \sin P_i = 0, \quad l = 1, \dots, L \quad (4.3)$$

for all the loops L_l . Eq.(4.2) means of course that for the singularity all the internal lines must go on-shell, but eq.(4.3) have more interesting consequences if we want the singularities to correspond to spacetime diagrams with the vertices (representing local interaction regions) being points in spacetime. Indeed in view of eq.(4.3) the diagram representing the singularity becomes closed if the internal line connecting two vertices is determined as

$$x_1^0 - x_2^0 = \alpha E, \quad x_1^1 - x_2^1 = \alpha \sin P, \quad (4.4)$$

instead of the usual expression [30], where P would appear instead of $\sin P$ on the r.h.s of the second equation in (4.4).[‡] Therefore the Landau diagrams in the magnon problem (i.e. spacetime diagrams representing the singularity) may have the same topology as the ordinary Landau diagrams but - unlike in the ordinary case - they cannot be interpreted as the propagation of on shell particles. (Note in particular, that the two metrics corresponding to the dispersion relations (2.3, 2.11) on the one hand and the one on the lines (4.4) on the other are different while in the ordinary case they are the same).[§]

[†]We consider the leading singularity of the diagram only

[‡]This subtle difference seems to have been unnoticed in the earlier works [15] [16].

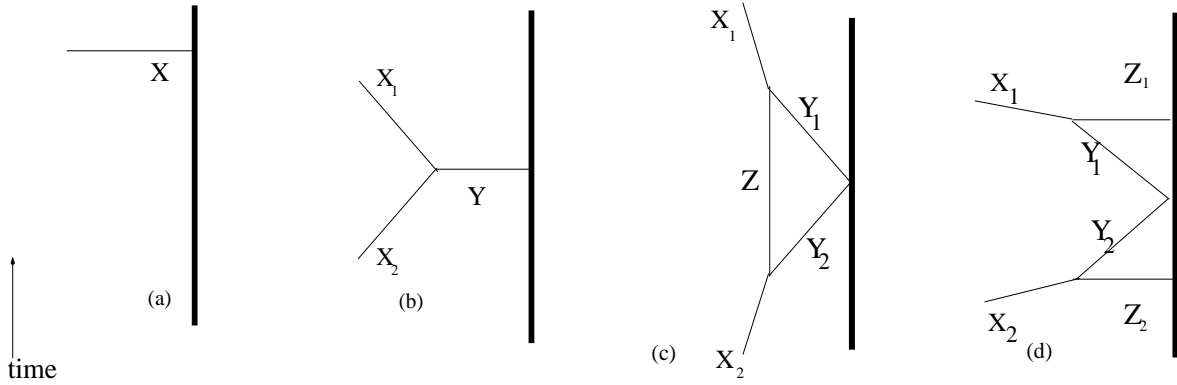
[§]It is interesting to note how the Landau equations change if one assumes the non relativistic propagator

$$G(E, P) = \frac{i}{E - \sqrt{Q^2 + 16g^2 \sin^2 \frac{P}{2}} + i\epsilon}$$

In the presence of reflecting boundaries eq.(4.2,4.3) remain valid and the only extra restriction is that at the vertex describing the reflection only the particle's energy is conserved while its momentum changes sign, i.e. at the reflection vertex the spectral parameter of the particle changes as $X^\pm \rightarrow -X^\mp$.

4.2 Landau diagrams for reflections on the $Y = 0$ brane

In the following we look for (boundary) Landau diagrams that could explain the first order poles listed in (3.4). To start with we give here a sample of 0, 1 and 2 loop diagrams used earlier to describe several poles in various reflection amplitudes of the boundary sine-Gordon model [26] [27]:



Here we adapted the diagrams to the magnon problem by characterizing the magnon(bound states) with their spectral parameters. At the bulk vertices energy, momentum and $U(1)$ charge are conserved and these are so restrictive, that the $(X, Y$ and $Z)$ spectral parameters of the three particles joined by the vertex should be related to each other by one of the instead of $\Pi(E, P)$ for (multi)magnons. In this case one finds the loop equations

$$\sum_{i \in L_l} \alpha_i = 0, \quad -4g \sum_{i \in L_l} \alpha_i v(P_i) = 0, \quad l = 1, \dots, L$$

where $v(P_i)$ is the particle's velocity, eq.(2.12). These equations admit no obvious space time interpretation in spite of the clear physical meaning of the second set.

following ‘vertex conditions’ [15][16]

$$X^- = Z^-, \quad X^+ = Y^+, \quad Z^+ = Y^- \quad (4.5)$$

$$X^+ = Z^+, \quad X^- = Y^-, \quad Z^- = Y^+ \quad (4.6)$$

$$X^- = Z^-, \quad X^+ = \frac{1}{Y^-}, \quad Z^+ = \frac{1}{Y^+} \quad (4.7)$$

$$X^+ = Z^+, \quad X^- = \frac{1}{Y^+}, \quad Z^- = \frac{1}{Y^-} \quad (4.8)$$

The question which of these and in what situation may correspond to bound state poles of the bulk S matrix is discussed at length in [15][16]. Because of the various possibilities to every diagram one should also tell the vertex conditions chosen at the bulk vertices. Therefore the following algorithm is devised to check all the candidate diagrams:

1. choose one of the admissible vertex conditions at every bulk vertex,
2. impose the reflection condition at the vertex on the boundary,
3. check that as a result one gets indeed the reflection of the external legs $X_1^\mp = -X_2^\pm$,
4. check whether Q for the internal lines - as determined from (2.9) - is physical ($Q \geq 0$),
5. check that at all (bulk) vertices the particles are indeed heading towards/away from the boundary by comparing their velocities to (3.10),
6. check whether the internal reflection/scattering amplitudes have zeroes or poles to modify the naive counting of the degree of the pole ($N - 2L$).

In checking the last three points it is assumed of course that we are at the pole in question, i.e. the incoming X_2^\pm is determined by the location of the pole (3.4) and by (2.9).

On several diagrams of the figure there are horizontal lines, corresponding to $E = 0$ magnon(bound states) according to (4.4). The spectral parameters of such a particle satisfy

$$(X^+ - X^-)(1 + \frac{1}{X^+ X^-}) = 0. \quad (4.9)$$

The $X^+ = X^-$ solution gives vanishing $Q = 0$, therefore we must choose the other, and this, when combined with (2.9) gives

$$X^- + \frac{1}{X^-} = -\frac{i}{2g}Q, \quad X^+ + \frac{1}{X^+} = \frac{i}{2g}Q \quad (4.10)$$

The fact that this X^- is different from the ones in (3.4) indicates that no poles may correspond to a particle “standing perpendicular” to the boundary (i.e. diagram (a) is ruled out). This may be understood in the following heuristic way: recently, in ordinary boundary integrable QFTs, it was shown, that the existence of a pole corresponding to a particle standing perpendicular to the boundary is related to some fields developing non trivial vacuum expectation values (vev) [31]. Assuming this relation exists also in the field theory underlying the magnon problem the absence of this pole is in accord with the residual supersymmetry that rules out non trivial vev-s.

Diagram (b) is also ruled out: using any of the vertex conditions and insisting on the reflection of the external leg $X_1^\mp = -X_2^\pm$ gives a system of equations that can simultaneously be satisfied only if X_2^\pm satisfy additional requirements (like $X_2^+ = -X_2^-$); which the actual solutions (3.6,3.7) fail to fulfill. A similar - but more involved - argument rules out diagram (d).

The triangle diagram (diagram (c)), providing naively a first order pole, is interesting as it contains a “vertical” line i.e. one which runs parallel to the boundary. The physically interesting value for the momentum of the corresponding particle is $P = \pm\pi^\P$, and this gives for the spectral parameters $Z^+ = -Z^-$. (It is shown in the following, that the actual solutions of the first three steps of the algorithm outlined earlier indeed contain such a line). Then it is interesting to note that eq.(4.4) can obviously be satisfied since $Y_1^\pm = -Y_2^\mp$ implies $P_1^Y = -P_2^Y$ and $E_1^Y = E_2^Y$, thus choosing $\alpha_1 = \alpha_2 = \alpha$ solves the second equation in (4.4) for any value of α , while the first requires only $2\alpha E_1^Y + \alpha_3 E^Z = 0$.

The outcome of the study of the various triangle diagrams is summarized in Table 1. Here we collected only those that after imposing the reflection at the vertex on the boundary ($Y_1^\pm = -Y_2^\mp$) lead to the reflection of the external legs $X_1^\pm = -X_2^\mp$. For all of them one also gets $Z^+ = -Z^-$ as promised earlier. In each case the vertex conditions used at the upper (lower) vertices on diagram (c) are listed first (second). The second column contains the Q values for the internal lines determined from the conservation equations and from X_2^\pm being determined by the pole condition (3.4) and (2.9):

$$X_2^- + \frac{1}{X_2^-} = i\frac{m}{2g}, \quad X_2^+ + \frac{1}{X_2^+} = i\frac{2Q+m}{2g}, \quad m = -(Q-1), \dots, -1, 1, 2, \dots \quad (4.11)$$

The second and fourth possibilities are ruled out since for the Y lines here we find the unphysical value $Q_{Y_1} = Q_{Y_2} \equiv Q_Y < 0$. The third possibility is acceptable for negative

^{\P}The $P = 0$ other solution leads to $Z^+ = Z^-$ and $Q = 0$, thus we discard it.

Table 1: The four triangle diagrams leading to the reflection of the external legs.

vertex conditions	Q values	reflecting magnon
$X_1^- = Z^-, X_1^+ = \frac{1}{Y_1^-}, Z^+ = \frac{1}{Y_1^+}$ $X_2^+ = Z^+, X_2^- = \frac{1}{Y_2^+}, Z^- = \frac{1}{Y_2^-}$	$Q_Z = 2Q + m$ $Q_Y = Q + m$	$Y_2^+ + \frac{1}{Y_2^+} = \frac{im}{2g}$ $Y_2^- + \frac{1}{Y_2^-} = -\frac{i}{2g}(2Q + m)$
$X_1^+ = Z^+, X_1^- = \frac{1}{Y_1^+}, Z^- = \frac{1}{Y_1^-}$ $X_2^- = Z^-, X_2^+ = \frac{1}{Y_2^-}, Z^+ = \frac{1}{Y_2^+}$	$Q_Y = -(Q + m)$	
$X_1^+ = Z^+, X_1^- = Y_1^-, Z^- = Y_1^+$ $X_2^- = Z^-, X_2^+ = Y_2^+, Z^+ = Y_2^-$	$Q_Z = -m$ $Q_Y = Q + m$	$Y_2^+ + \frac{1}{Y_2^+} = \frac{i}{2g}(2Q + m)$ $Y_2^- + \frac{1}{Y_2^-} = -\frac{im}{2g}$
$X_1^- = Z^-, X_1^+ = Y_1^+, Z^+ = Y_1^-$ $X_2^+ = Z^+, X_2^- = Y_2^-, Z^- = Y_2^+$	$Q_Y = -(Q + m)$	

m -s: $m \in -(Q-1), \dots, -1$. In the third column the Y_2^\pm spectral parameters of the internal particle reflecting on the boundary are collected. These are useful when checking whether the internal reflection amplitude has a zero or a pole which would modify the naive degree of the diagram. To do this one has to compare these values to the ones in (3.4) and (3.5) where the substitution $Q \rightarrow Q_Y$ is made. From this it turns out that for the first possibility the internal reflection amplitude has a zero at this particular values of the spectral parameters thus rendering the diagram *finite*, while for third possibility there is a pole so that the diagram gives finally a *second order pole*. Thus the first diagram can not be used to explain the poles in (3.4) while the third may perhaps be combined with other diagrams giving also second order poles.

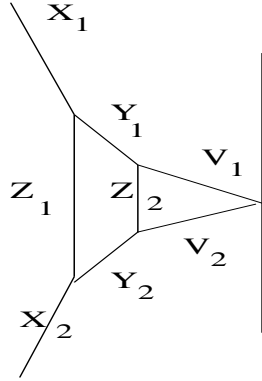
Finally, to complete the study of the triangle diagrams we give here the velocity of the particle Y_2^\pm at the lower vertex on diagram (c) for the first and third cases^{||}:

$$\text{first : } v(Y_2) = \frac{1 - \frac{X_2^+}{X_2^-}}{\frac{1}{X_2^-} - X_2^+}, \quad \text{third : } v(Y_2) = \frac{1 - \frac{X_2^+}{X_2^-}}{X_2^+ - \frac{1}{X_2^-}}. \quad (4.12)$$

Obviously only one of them can have the same sign as $v(X_2)$, (3.10), and using the actual values of X_2^\pm in (3.6,3.7) reveals that it is the *first* diagram where this happens. Therefore the third diagram is also ruled out and we conclude that there is no simple triangle diagram that could be used to explain the first order poles (3.4).

^{||}the velocity of the particle Y_1 at the upper vertex is automatically opposite to this one

The way out is to consider appropriate two loop diagrams obtained by combining in a certain sense the simple triangles of Table 1 where the zero of the reflection amplitude renders the diagram first order. The diagram that works



has the same upper and lower vertex conditions along the longer vertical line as the first possibility in the table (with $Z \rightarrow Z_1$) while along the shorter vertical line these conditions are given by the fourth possibility there (with the substitutions $X_{1,2} \rightarrow Y_{1,2}$, $Y_{1,2} \rightarrow V_{1,2}$, $Z \rightarrow Z_2$). Imposing the $V_1^\pm = -V_2^\mp$ reflection condition results in $X_1^\pm = -X_2^\mp$ and for the internal particles the Q values turn out to be:

$$Q_{Z_1} = 2Q + m, \quad Q_{Y_2} = Q_{Y_1} = Q + m, \quad Q_{V_2} = Q_{V_1} = Q, \quad Q_{Z_2} = m, \quad (4.13)$$

which, for $m \geq 1$, are physically acceptable. Furthermore for the spectral parameters of the reflecting magnon one finds

$$V_2^+ + \frac{1}{V_2^+} = -\frac{im}{2g}, \quad V_2^- + \frac{1}{V_2^-} = -\frac{i}{2g}(2Q + m), \quad (4.14)$$

indicating that the internal reflection has indeed a zero at this particular point. Since naively the diagram would give a second order pole the existence of this zero reduces it to a first order one. The velocity $v(Y_2)$ is the same as in the first case of (4.12), while

$$v(V_2) = -\frac{1}{v(X_2)} = -\frac{N}{i4gL} \quad (4.15)$$

showing that at the two lower vertices the velocity of the internal particles points along that of the incoming one thus the diagram is consistent.

This way it is demonstrated that (in principle at least) the first order poles, eq.(3.4), can be explained in terms of Landau type diagrams. The systematic analysis - that would require a detailed study of all possible diagrams and also would involve the the comparison

of the residues of the poles of the Landau diagrams and that of the reflection amplitude (3.3) - is beyond the scope of the present paper. Similarly the analysis of the poles of the more complicated amplitudes describing (multi)magnon reflections on the $Z = 0$ brane [9] [13] is left for a future investigation.

5 Boundary state formalism in the magnon problem

In this section we generalize the boundary state formalism - originally worked out for relativistic integrable boundary theories by Ghoshal and Zamolodchikov [17] - for the case of magnon reflections.

5.1 Summary of boundary state formalism for relativistically invariant boundary theories

Ghoshal and Zamolodchikov showed that the relativistically invariant integrable boundary theories admit two *equivalent* Hamiltonian descriptions. In the so called “open channel” there is a boundary (represented by the operator B) and the bulk particles (represented by the ZF operators $A_j^\dagger(\theta)$) reflect non trivially on this boundary

$$A_i^\dagger(\theta)B = R_i^j(\theta)A_j^\dagger(-\theta)B \quad (5.1)$$

(θ denotes the rapidity of the particle). Exchanging the time and space coordinates in the Euclidean version of the theory (i.e. doing a double Wick rotation) one obtains the so called “closed channel” which is nothing but the periodic bulk model without any boundaries. In this channel there is a special state, the boundary state $\langle B|$, that carries all the information about the boundary. The requirement that determines the boundary state is that the correlation functions computed in the two channels should be identical:

$$\langle O_1(x_1, y_1) \dots O_N(x_N, y_N) \rangle = \frac{{}_B\langle 0 | \mathcal{T}_y O_1(x_1, y_1) \dots O_N(x_N, y_N) | 0 \rangle_B}{{}_B\langle 0 | 0 \rangle_B}, \quad (5.2)$$

$$\langle O_1(x_1, y_1) \dots O_N(x_N, y_N) \rangle = \frac{\langle B | \mathcal{T}_x (O_1(x_1, y_1) \dots O_N(x_N, y_N)) | 0 \rangle}{\langle B | 0 \rangle} \quad (5.3)$$

The boundary state is a kind of coherent state that - in the simplest case - can be written in terms of the *out*-states as

$$\langle B| = \langle 0| \left(1 + \int_0^\infty d\theta K^{ab}(\theta) A_a(-\theta) A_b(\theta) + \dots \right), \quad (5.4)$$

where dots stand for the contribution containing higher number of particles. The values of the amplitude describing the two particle contribution to the boundary state $K^{ab}(\theta)$ at negative real θ are interpreted as the coefficients of expansion of $\langle B |$ in terms of the *in*-states $\langle 0 | A_a(\theta) A_b(-\theta)$, $\theta > 0$:

$$\langle B | = \langle 0 | \left(1 + \int_0^\infty d\theta K^{ab}(-\theta) A_a(\theta) A_b(-\theta) + \dots \right). \quad (5.5)$$

Since $A_a(\theta) A_b(-\theta)$ and $A_a(-\theta) A_b(\theta)$ are related by the S matrix

$$A_a(\theta) A_b(-\theta) = S_{ab}^{cd}(2\theta) A_d(-\theta) A_c(\theta), \quad (5.6)$$

the consistency of the two ways of writing the first two elements of the boundary state requires

$$K^{dc}(\theta) = K^{ab}(-\theta) S_{ab}^{cd}(2\theta). \quad (5.7)$$

The importance of this equation is that it provides the boundary crossing condition (BCC) for the reflection amplitudes once K^{ab} and R_i^j are related. This link is provided by the reduction formulae from which it follows, that $K^{ab}(\theta)$ is related to the (analytical continuation of the) reflection amplitude as

$$K^{ab}(\theta) = R_a^b(i\frac{\pi}{2} - \theta). \quad (5.8)$$

Furthermore in [17] it is shown that the boundary Yang-Baxter equations when combined with (5.8) and the unitarity and crossing properties of the bulk S matrix are sufficient to guarantee that

$$[K(\theta), K(\theta')] = 0, \quad \text{where} \quad K(\theta) = K^{ab}(\theta) A_a(-\theta) A_b(\theta), \quad (5.9)$$

and as a consequence the boundary state can be written as

$$\langle B | = \langle 0 | \exp \left(\frac{1}{2} \int_{-\infty}^{\infty} K(\theta) d\theta \right), \quad (5.10)$$

without any ordering problems.

5.2 Boundary state in the magnon problem

There are two major problems one has to solve when trying to implement the boundary state formalism in the magnon problem: namely the theory underlying magnon scattering/reflections is *not relativistically invariant* and also the analogues of the reduction formulae are missing.

As a consequence of the non-relativistic nature of the magnon theory the so called “mirror” magnon theory (which is obtained by the double Wick rotation and is defined in the closed channel) is not equivalent to the original (open channel) one. The mirror magnon theory in the bulk is worked out in details in [18]. There it is shown that the momenta and energies of the magnon (p, E) and mirror magnon (\tilde{p}, \tilde{E}) are related through the analytic continuations:

$$p \rightarrow 2i \operatorname{arcsinh}\left(\frac{\sqrt{1 + \tilde{p}^2}}{4g}\right) = i\tilde{E}, \quad E = \sqrt{1 + 16g^2 \sin^2 \frac{p}{2}} \rightarrow i\tilde{p}. \quad (5.11)$$

Realizing that the dispersion relation (2.1) describes a complex torus [6, 8] the magnon energy, momentum or equivalently the spectral parameters x^\pm can be expressed in terms of Jacobi elliptic functions:

$$p(z) = 2\operatorname{am}(z), \quad \sin \frac{p(z)}{2} = \operatorname{sn}(z, k) \equiv \operatorname{sn}(z), \quad E(z) = \operatorname{dn}(z, k) \equiv \operatorname{dn}(z), \quad (5.12)$$

$$x^\pm(z) = \frac{1}{4g} \left(\frac{\operatorname{cn}(z, k)}{\operatorname{sn}(z, k)} \pm i \right) (1 + \operatorname{dn}(z, k)). \quad (5.13)$$

Here the elliptic modulus $k^2 = -16g^2 \in \mathbb{R}$ is fixed in a given theory thus p, E or x^\pm can be regarded as functions of the complex parameter z called “generalized rapidity”. The two cycles of the rapidity torus can be described by the shifts $z \rightarrow z \pm 2\omega_1, z \rightarrow z \pm 2\omega_2$ with

$$\omega_1 = 2K(k^2), \quad \omega_2 = 2iK(1 - k^2) - 2K(k^2), \quad (5.14)$$

where $K(k^2)$ is the complete elliptic integral of the first kind. For our k^2 -s $\operatorname{Im} \omega_1 = 0 = \operatorname{Re} \omega_2$. To make contact with relativistic theories we consider the limit $g \rightarrow \infty$ when the periods of the torus have the following behaviour

$$\omega_1 \rightarrow \frac{\log g}{2g}, \quad \omega_2 \rightarrow i\frac{\pi}{4g}. \quad (5.15)$$

Rescaling z as $z \rightarrow z/(4g)$ and the momentum as $p \rightarrow p/(2g)$ keeps the range of $\operatorname{Im}(z)$ finite and converts also the dispersion relation (2.1) to the relativistic form $E^2 - p^2 = 1$ thus showing that the variable z indeed plays the role of θ (because $p = \sinh z$). Furthermore in this limit the torus degenerates into the strip $-\pi < \operatorname{Im}(z) < \pi$ and $-\infty < \operatorname{Re}(z) < \infty$. In [18] it is shown that the magnon S -matrix (2.13) admits an analytic continuation $\mathcal{S}(z_1, z_2)$ to the entire rapidity torus.

The z -torus can also be used to describe the mirror model. The trick is to realize that the double Wick rotation can be implemented [18] by the shift $z \rightarrow \tilde{z} + \frac{\omega_2}{2}$ since then

$$\tilde{p} = -i \operatorname{dn}(\tilde{z} + \frac{\omega_2}{2}, k) = \sqrt{k'} \frac{\operatorname{sn}(\tilde{z})}{\operatorname{cn}(\tilde{z})}, \quad \tilde{E} = 2 \operatorname{arccoth} \frac{\sqrt{k'}}{\operatorname{dn}(\tilde{z})}, \quad (5.16)$$

i.e. \tilde{p} is real for real \tilde{z} . Furthermore the range $-\infty < \tilde{p} < \infty$ corresponds to $-\omega_1/2 < \tilde{z} < \omega_1/2$. Note also that this shift is completely analogous to $\theta \rightarrow \theta + i\frac{\pi}{2}$ connecting the rapidities of particles in the open and closed channels in case of relativistic theories. According to Arutyunov and Frolov the mirror magnon's scattering matrix $\tilde{\mathcal{S}}(\tilde{z}_1, \tilde{z}_2)$ is related to the magnon S matrix as [18]:

$$\tilde{\mathcal{S}}(\tilde{z}_1, \tilde{z}_2) = \mathcal{S}(\tilde{z}_1 + \frac{\omega_2}{2}, \tilde{z}_2 + \frac{\omega_2}{2}). \quad (5.17)$$

5.2.1 Boundary state for magnon reflections on the $Y = 0$ brane

The simple description of boundary state given by (5.4,5.5) can immediately be generalized to magnons reflecting on the $Y = 0$ brane since this problem contains no boundary degrees of freedom (and no boundary bound states). Restricting our attention to magnon reflection matrices corresponding to a single copy of the centrally extended $su(2|2)$ algebra** the analogue of (5.1) is

$$A_i^\dagger(z)B = R_i^j(z)A_j^\dagger(-z)B \quad (5.18)$$

where the indices $i, j = 1 \dots 4$. Symmetry considerations restrict the explicit form of the reflection matrix as $R_i^j(z) = R_0(z) \operatorname{diag}(e^{-ip(z)}, -1, 1, 1)$ [9] [14]. Denoting the ZF operators for the mirror magnons as $\tilde{A}_a(\tilde{z})$ the boundary state can be written in terms of the generalized (shifted) rapidity as

$$\langle B| = \langle 0| \left(1 + \int_0^{\omega_1/2} d\tilde{z} \rho(\tilde{z}) K^{ab}(\tilde{z}) \tilde{A}_a(-\tilde{z}) \tilde{A}_b(\tilde{z}) + \dots \right) \quad (5.19)$$

where $\rho(\tilde{z})$ is the density of states that plays no role in our considerations. The consistency condition of the two ways of expressing the boundary state has the form now:

$$K^{dc}(\tilde{z}) = K^{ab}(-\tilde{z}) \tilde{S}_{ab}^{cd}(\tilde{z}, -\tilde{z}), \quad (5.20)$$

i.e. naturally it contains the S matrix of the mirror model.

**with the understanding that the complete reflection matrix is the tensor product of two such R -s

(5.20) can be interpreted as the BCC for the reflection matrix if K^{ab} and R_i^j are somehow related. In the lack of reduction formulae in the magnon model we determine a relation between them by demanding that the $z \rightarrow z + \frac{\omega_2}{2}$ continuation of the boundary Yang-Baxter equation for R_i^j

$$R_j^k(z_2)S_{ik}^{lm}(z_1, -z_2)R_l^n(z_1)S_{mn}^{wv}(-z_2, -z_1) = S_{ij}^{mn}(z_1, z_2)R_m^q(z_1)S_{nq}^{uv}(z_2, -z_1)R_u^w(z_2), \quad (5.21)$$

when combined with the unitarity and crossing properties of the magnon S matrix

$$\mathcal{S}_{12}(z_1, z_2)\mathcal{S}_{21}(z_2, z_1) = \mathbb{I}, \quad \mathbb{C}^{-1}\mathcal{S}_{12}^{tr}(z_1, z_2)\mathbb{C}\mathcal{S}_{12}(z_1, z_2 - \omega_2) = \mathbb{I}, \quad (5.22)$$

and with the AF relation (5.17) between the magnon and mirror magnon's S matrices should guarantee that

$$[K(z), K(z')] = 0, \quad \text{where} \quad K(z) = K^{ab}(z)\tilde{A}_a(-z)\tilde{A}_b(z). \quad (5.23)$$

After a not very illuminating computation one finds that this condition is met if

$$K^{ab}(z) = \mathbb{C}^{ac}R_c^b\left(\frac{\omega_2}{2} - z\right) \quad (5.24)$$

where \mathbb{C} is the charge conjugation matrix. Note the complete analogy of this equation to (5.8) obtained by using the reduction formulae.

Since (5.24) is obtained by requiring (5.23) the boundary state has a similar exponential form as in the BIQFT case (5.10). Furthermore, plugging (5.24) into (5.20), using eq.(5.17) and continuing back by substituting $z \rightarrow u + \frac{\omega_2}{2}$ gives

$$\mathbb{C}^{ac_1}R_{c_1}^b(\omega_2 + u)S_{ab}^{cd}(u + \omega_2, -u) = \mathbb{C}^{dm}R_m^c(-u). \quad (5.25)$$

Using the unitarity of the reflection matrix $R(u)R(-u) = 1$ and the $S(z_1 + \omega_2, z_2 + \omega_2) = S(z_1, z_2)$ “translational invariance” property of the magnon S matrix this last equation can be converted to

$$\mathbb{C}^{ac_1}R_{c_1}^b(\omega_2 + u)S_{ab}^{cd}(u, -u - \omega_2)R_c^n(u) = \mathbb{C}^{dn}, \quad (5.26)$$

which, in the light of $x^\pm(u + \omega_2) = (x^\pm(u))^{-1}$, is the BCC in [14].

5.2.2 Boundary state for magnon reflections on the $Z = 0$ brane

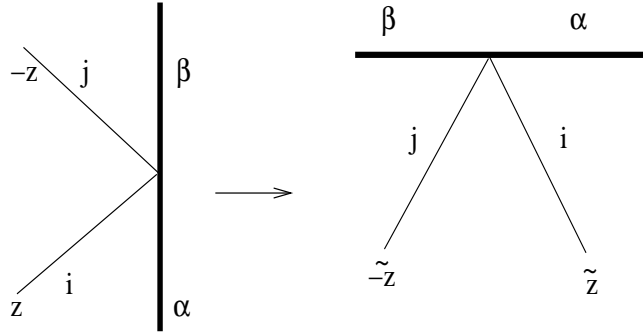
The essentially new feature of the magnon reflection on the $Z = 0$ brane is the presence of boundary degrees of freedom [9]. This means that in this case the boundary is not a

singlet but belongs to the same fundamental representation of the symmetry algebra as the magnons themselves (albeit with a slightly different relation between the central charges). As a consequence in a reflection process also the boundary may change and eq.(5.18) generalizes to

$$A_i^\dagger(z)B_\alpha = R_{i\alpha}^{j\beta}(z)A_j^\dagger(-z)B_\beta \quad (5.27)$$

where B_α $\alpha = 1 \dots 4$ are the operators representing the $Z = 0$ boundary. The explicit form of the reflection matrix as obtained from imposing the symmetry requirement is given in eq.(3.32) and (3.33) of [14].

Looking at the graphical representation of the magnon reflection together with its image in the closed channel



it is easy to argue that in the closed channel one indeed obtains the periodic mirror magnon model but with several sectors characterized by the boundary degrees of freedom α, β . In each sector there is a boundary state

$$\langle B_{\alpha\beta} | = \langle 0 | (\delta_{\alpha\beta} + \int_0^{\omega_1/2} d\tilde{z} \rho(\tilde{z}) K^{\alpha\beta}(\tilde{z}) + \int_0^{\omega_1/2} d\tilde{z} \rho(\tilde{z}) \int_0^{\omega_1/2} d\tilde{w} \rho(\tilde{w}) K^{\alpha\gamma}(\tilde{z}) K^{\gamma\beta}(\tilde{w}) + \dots) \quad (5.28)$$

where

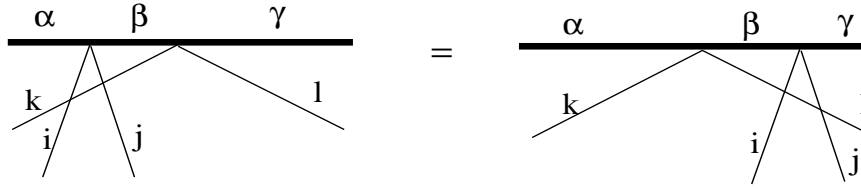
$$K^{\alpha\beta}(\tilde{z}) = K^{ab\alpha\beta}(\tilde{z}) \tilde{A}_a(-\tilde{z}) \tilde{A}_b(\tilde{z}). \quad (5.29)$$

The consistency condition of the two ways of expressing the boundary state has the form now:

$$K^{dc\alpha\beta}(\tilde{z}) = K^{ab\alpha\beta}(-\tilde{z}) \tilde{S}_{ab}^{cd}(\tilde{z}, -\tilde{z}) \quad (5.30)$$

Note that both the boundary state (5.28) and the consistency condition (5.30) are simply obtained by “decorating” with the indices $\alpha \beta$ the corresponding expressions for the $Y = 0$ case, eq.(5.19-5.20).

Recalling that the analytically continued boundary Yang Baxter equation - that graphically can be represented as -



contains a summation over the intermediate boundary degree of freedom β makes it plausible that demanding the vanishing of

$$[K(\tilde{z}), K(\tilde{w})]^{\alpha\beta} \equiv K^{\alpha\gamma}(\tilde{z})K^{\gamma\beta}(\tilde{w}) - K^{\alpha\gamma}(\tilde{w})K^{\gamma\beta}(\tilde{z}) = 0 \quad (5.31)$$

when combined with eq.(5.17) and eq.(5.22) gives indeed a useful relation between the reflection matrix and the coefficient of the two particle contribution to the boundary state. This way one obtains

$$K^{ab\alpha\beta}(z) = \mathbb{C}^{ac} R_{c\alpha}^{b\beta} \left(\frac{\omega_2}{2} - z \right) \quad (5.32)$$

in complete analogy to eq.(5.24) and (5.8). Proceeding in the same way as in the case of reflections on the $Y = 0$ brane - i.e. continuing back by the substitution $z \rightarrow u + \frac{\omega_2}{2}$ and exploiting the unitarity of $R_{b\beta}^{a\alpha}$ and the translational symmetry of the magnon scattering matrix - one can show that the consistency condition, eq.(5.30), becomes indeed the BCC for the $Z = 0$ brane, eq.(4.24) of [14].

6 Conclusions

In this paper two problems related to reflections of (multi)magnons in AdS/CFT are discussed. In the first problem, aimed at giving an interpretation in terms of Landau equations and Landau diagrams of the poles of the reflection amplitudes that do not correspond to boundary bound states we pointed out that the derivation of Landau equations for the magnon problem requires the knowledge of the free propagator for the (multi)magnons. Using an appropriate candidate for this propagator the Landau equations were derived and some differences to the ordinary case were pointed out. As a result of these differences the singularities of the the magnon reflection/scattering amplitudes may be interpreted in terms of space time (Landau) diagrams, but - unlike in the relativistic case - these diagrams do not correspond to the propagation of on shell particles. In addition a detailed study of Landau

diagrams describing the first order poles of the magnon reflection amplitudes on the $Y = 0$ brane is presented.

The boundary state formalism originally worked out for relativistic boundary integrable models by Ghoshal and Zamolodchikov is successfully generalized to magnon reflections on both the $Y = 0$ and the $Z = 0$ branes. This way a new derivation is obtained of the boundary crossing condition and this is interesting, as this condition plays an important role in determining the scalar factor of the reflection amplitudes which is left undetermined by the symmetry considerations. In addition the boundary states constructed may be useful to investigate the finite size effects (TBA) of magnon reflections.

Acknowledgments

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